

## Module 3 Lesson 1

1. Apply the properties of exponents to rewrite each expression in the form  $kx^n$ , where  $n$  is an integer and  $x \neq 0$ .

a.  $(2x^3)(3x^5)(6x)^2$

d.  $5(x^3)^{-3}(2x)^{-4}$

b.  $\frac{3x^4}{(-6x)^{-2}}$

e.  $\left(\frac{x^2}{4x^{-1}}\right)^{-3}$

c.  $\frac{x^{-3}x^5}{3x^4}$

2. Apply the properties of exponents to verify that each statement is an identity.

a.  $\frac{2^{n+1}}{3^n} = 2\left(\frac{2}{3}\right)^n$  for integer values of  $n$

b.  $3^{n+1} - 3^n = 2 \cdot 3^n$  for integer values of  $n$

c.  $\frac{1}{(3^n)^2} \cdot \frac{4^n}{3} = \frac{1}{3}\left(\frac{2}{3}\right)^{2n}$  for integer values of  $n$

3. Jonah was trying to rewrite expressions using the properties of exponents and properties of algebra for nonzero values of  $x$ . In each problem, he made a mistake. Explain where he made a mistake in each part, and provide a correct solution.

Jonah's Incorrect Work

a.  $(3x^2)^{-3} = -9x^{-6}$

b.  $\frac{2}{3x^{-5}} = 6x^5$

c.  $\frac{2x-x^3}{3x} = \frac{2}{3} - x^3$

4. If  $x = 5a^4$  and  $a = 2b^3$ , express  $x$  in terms of  $b$ .

## Module 3 Lesson 2

1. Write the following numbers used in these statements in scientific notation. (Note: Some of these numbers have been rounded.)

a. The density of helium is 0.0001785 gram per cubic centimeter.

f. One cubic inch is 0.0000214 cubic yard.

b. The boiling point of gold is 5,200°F.

g. Earth's population in 2012 was 7,046,000,000 people.

c. The speed of light is 186,000 miles per second.

h. Earth's distance from the sun is 93,000,000 miles.

d. One second is 0.000278 hour.

i. Earth's radius is 4,000 miles.

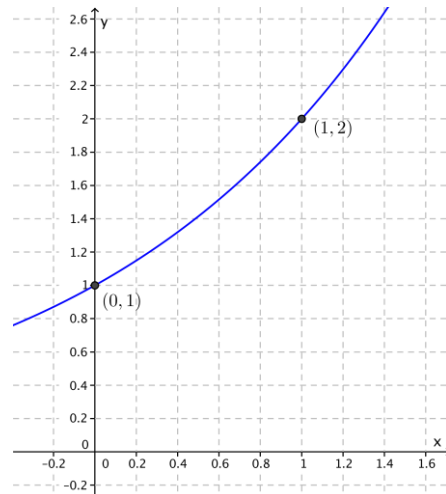
e. The acceleration due to gravity on the sun is 900 ft/s<sup>2</sup>.

j. The diameter of a water molecule is 0.000000028 cm.

2. Write the following numbers in decimal form. (Note: Some of these numbers have been rounded.)
- A light year is  $9.46 \times 10^{15}$  m.
  - Avogadro's number is  $6.02 \times 10^{23}$  mol<sup>-1</sup>.
  - The universal gravitational constant is  $6.674 \times 10^{-11}$  N  $\left(\frac{\text{m}}{\text{kg}}\right)^2$ .
  - Earth's age is  $4.54 \times 10^9$  years.
  - Earth's mass is  $5.97 \times 10^{24}$  kg.
  - A foot is  $1.9 \times 10^{-4}$  mile.
  - The population of China in 2014 was  $1.354 \times 10^9$  people.
  - The density of oxygen is  $1.429 \times 10^{-4}$  gram per liter.
  - The width of a pixel on a smartphone is  $7.8 \times 10^{-2}$  mm.
  - The wavelength of light used in optic fibers is  $1.55 \times 10^{-6}$  m.
3. Perform the following calculations without rewriting the numbers in decimal form.
- $(2.5 \times 10^4) + (3.7 \times 10^3)$
  - $(6.9 \times 10^{-3}) - (8.1 \times 10^{-3})$
  - $(6 \times 10^{11})(2.5 \times 10^{-5})$
  - $\frac{4.5 \times 10^8}{2 \times 10^{10}}$

### Module 3 Lesson 3

1. Rewrite in radical form. If the number is rational, write it without using radicals.
- $6^{\frac{3}{2}}$
  - $\left(\frac{1}{2}\right)^{\frac{1}{4}}$
  - $3(8)^{\frac{1}{3}}$
  - $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$
  - $81^{-\frac{1}{4}}$
2. Rewrite the following expressions in exponent form.
- $\sqrt{5}$
  - $\sqrt[3]{5^2}$
  - $\sqrt{5^3}$
  - $(\sqrt[3]{5})^2$
3. Use the graph of  $f(x) = 2^x$  shown to the right to estimate the following powers of 2.
- $2^{\frac{1}{4}}$
  - $2^{\frac{2}{3}}$
  - $2^{\frac{3}{4}}$
  - $2^{0.2}$
  - $2^{1.2}$
  - $2^{-\frac{1}{5}}$



## Module 3 Lesson 4

1. Rewrite each expression so that each term is in the form  $kx^n$ , where  $k$  is a real number,  $x$  is a positive real number, and  $n$  is a rational number.

a.  $x^{-\frac{2}{3}} \cdot x^{\frac{1}{3}}$

b.  $2x^{\frac{1}{2}} \cdot 4x^{-\frac{5}{2}}$

c.  $\frac{10x^{\frac{1}{3}}}{2x^2}$

d.  $(3x^{\frac{1}{4}})^{-2}$

e.  $\sqrt[3]{\frac{27}{x^6}}$

f.  $\sqrt[3]{x} \cdot \sqrt[3]{-8x^2} \cdot \sqrt[3]{27x^4}$

## Module 3 Lesson 5

1. Use properties of exponents to rewrite the following expressions as a number or an exponential expression with only one exponent.

a.  $(2^{\sqrt{3}})^{\sqrt{3}}$

b.  $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$

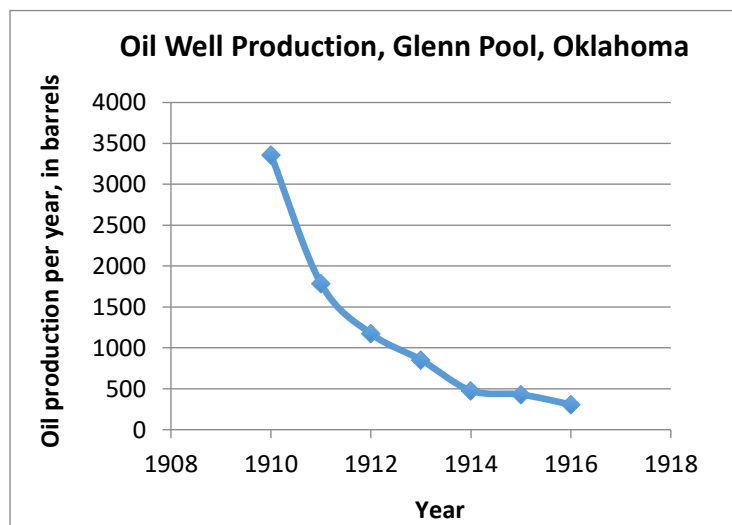
c.  $(3^{1+\sqrt{5}})^{1-\sqrt{5}}$

d.  $3^{\frac{1+\sqrt{5}}{2}} \cdot 3^{\frac{1-\sqrt{5}}{2}}$

e.  $3^{\frac{1+\sqrt{5}}{2}} \div 3^{\frac{1-\sqrt{5}}{2}}$

## Module 3 Lesson 6

1. The following graph shows the number of barrels of oil produced by the Glenn Pool well in Oklahoma from 1910 to 1916.



Source: Cutler, Willard W., Jr. Estimation of Underground Oil Reserves by Oil-Well Production Curves, U.S. Department of the Interior, 1924.

- Estimate the average rate of change of the amount of oil produced by the well on the interval  $[1910, 1916]$ , and explain what that number represents.
  - Estimate the average rate of change of the amount of oil produced by the well on the interval  $[1910, 1913]$ , and explain what that number represents.
  - Estimate the average rate of change of the amount of oil produced by the well on the interval  $[1913, 1916]$ , and explain what that number represents.
  - Compare your results for the rates of change in oil production in the first half and the second half of the time period in question in parts (b) and (c). What do those numbers say about the production of oil from the well?
  - Notice that the average rate of change of the amount of oil produced by the well on any interval starting and ending in two consecutive years is always negative. Explain what that means in the context of oil production.
2. The following table lists the number of hybrid electric vehicles (HEVs) sold in the United States between 1999 and 2013.

Year	Number of HEVs Sold in U.S.	Year	Number of HEVs Sold in U.S.
1999	17	2007	352,274
2000	9350	2008	312,386
2001	20,282	2009	290,271
2002	36,035	2010	274,210
2003	47,600	2011	268,752
2004	84,199	2012	434,498
2005	209,711	2013	495,685
2006	252,636		

Source: U.S. Department of Energy, Alternative Fuels and Advanced Vehicle Data Center, 2013.

- During which one-year interval is the average rate of change of the number of HEVs sold the largest? Explain how you know.
- Calculate the average rate of change of the number of HEVs sold on the interval  $[2003, 2004]$ , and explain what that number represents.
- Calculate the average rate of change of the number of HEVs sold on the interval  $[2003, 2008]$ , and explain what that number represents.
- What does it mean if the average rate of change of the number of HEVs sold is negative?